PAQ-003-1162001

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Seat No. ________

M. Sc. (Sem. II) Examination

August / September - 2020

Mathematics : CMT - 2001

(Algebra -II)

Faculty Code : 003

Subject Code : 1162001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Each question carries 14 marks.

1 Answer Any **Seven** short questions : **7×2=14**

(i) For a ring R, define R–module and give an example of an R–module.

(ii) Let M be an R–module. In standard notation, prove that $(-a)m = a(-m), \forall a \in R \; \& \; \forall m \in M$.

(iii) Let $f(x) = x^3 + 4x^2 - 11x + 13$. Prove that $f(x+1)$ is an irreducible polynomial over $Z[x]$.

(iv) Define finite field extension and give an example of finite extension.

(v) Write down all the roots of the polynomial $x^4 - 2 \in Q[x]$.

(vi) Write down the minimal polynomial of the number $\sqrt{2} + \sqrt{3}$ over Q.

(vii) Write down at least two irreducible polynomials of the ring $Z_2[x]$ whose degree is precisely two.

(viii) Give definition of an algebraic extension. Also give an example of an algebraic extension.

(ix) For a field extension $E|_F$, when we say E is finitely generated field over F? Also give definition of simple extension.

2 Attempt Any **Two** : **2×7=14**

(a) Let $E|_F$ and $K|_E$ both are finite extensions.

Prove that $K|_F$ is also a finite field extension.

(b) Let $p(x) \in F[x]$ be an irreducible polynomial and degree of $p(x) = n$. Let. $E|_F$ be an extension such that $\alpha \in E$ and α is a root of $p(x)$. Prove that $F[\alpha] = F(\alpha), [F(\alpha) : F] = n$ and $\{1, \alpha, \alpha^2, \ldots\ldots \alpha^{n-1}\}$ is a basis of $F(\alpha)$ over F.

(c) State and Prove Eisenstein Criterion.

3 Attempt Any **One** : **1×14=14**

(a) Let $E|_F$ and $K|_E$ both are finite separable extensions.

Prove that $K|_F$ is also a finite separable extension.

(b) (1) Let F be finite field. Prove that $F - \{0\}$ is a cyclic group under multiplication.

(2) Let F be a field and $F - \{0\}$ is a cyclic group under multiplication. Prove that F is a finite field.

(c) Let $E|_F$ be a finite extension. Prove that following statements are equivalent :

(1) $E = F(\propto)$, for some $\propto \in E$.

(2) There are only a finite number of sub fields of E containing F, as a sub field.

4 Attempt Any **two** : **2×7=14**

(a) Let $f(x) \in F[x]$ be an irreducible polynomial. Prove that α is a multiple root of $f(x)$ if and only if $f'(x) = 0$ (All the coefficients of $f'(x)$ are multiple of char F).

(b) Let p be a prime. Prove that $f(x) = x^{p-1} + x^{p-2} + \dots + x + 1 \in \mathbb{Z}[x]$ is an irreducible polynomial over $\mathbb{Q}[x]$.

(c) Let $f : M \to N$ be an R–homomorphism of R–modules. Prove that Ker f and $f(M)$ are R–sub modules of M and N respectively.

5 Attempt Any **Two** : **2×7=14**

(1) Let R be a ring and M be an R–module. Prove that M is a cyclic R–Module if and only if $M \cong \frac{R}{I}$, for some ideal I of R.

(2) Let $f : M \to N$ be an onto R–homomorphism of R–modules. Prove that $\frac{M}{Ker\ f} \cong N$.

(3) Let A, B, be R–sub modules of two R–modules M and N respectively. In standard notation, prove that $\frac{M \times N}{A \times B} \cong \frac{M}{A} \times \frac{N}{B}$.

(4) Let K be a field and char $K = p > 0$. Prove that K is a perfect field if and only if $K = K^p$, where $K^p = \{\alpha^p / \alpha \in K\}$.